
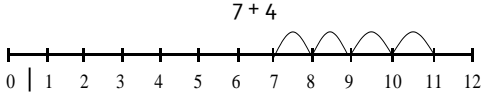
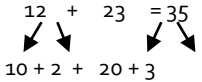
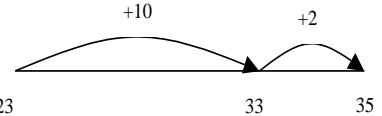
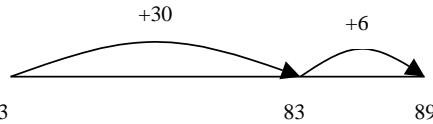
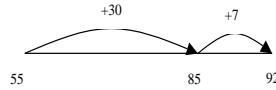
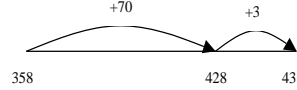
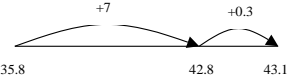


Addition Stage 1	Addition Stage 2	Addition Stage 3																								
<p>Pictures and symbols Sam has 3 marbles and his friend Tom has 4. How many marbles do they have altogether</p>  <p>$3 + 4 = \square$</p> <p>Number lines (numbered)</p>  <p>7 + 4</p> <p>Recording by - drawing jumps on prepared lines</p> <p> $\square = 3 + 4$ $3 + \square = 7$ $\square + 4 = 7$ $\square + \nabla = 7$ </p> <p> $7 = \square + 4$ $7 = 3 + \square$ $7 = \square + \nabla$ </p>	<p>Partition into tens and ones and recombine</p>  <p> $12 + 23 = 35$ $10 + 2 + 20 + 3$ </p> <p>add the tens $10 + 20 = 30$ add the ones $2 + 3 = 5$</p> <p>$30 + 5 = 35$</p> <p>Partition the smaller number regardless to where it is.</p> <p> $23 + 12 = 23 + 10 + 2$ $= 33 + 2$ $= 35$ </p> <p>Children use a blank number line and write their own digits:</p>  <p>23 33 35</p>	<p>Partition into tens and ones and recombine Partition the second number only (smallest number) e.g.</p> <p> $36 + 53 = 53 + 36 = 53 + 30 + 6$ $= 83 + 6$ $= 89$ </p>  <p>53 83 89</p> <p>Adding a near multiple of 10 to a two-digit number Continue as in stage 2 but with two digit numbers e.g. $35 + 19$ is the same as $35 + 20 - 1$.</p> <p>PENCIL AND PAPER PROCEDURES 83 (with and without carrying) expanded method moving to standard procedure e.g. $+48$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;">Carrying over happens at the bottom.</div> <p style="text-align: right;">131 1</p>																								
<p>Addition Stage 4</p> <p>Partition into tens and ones and recombine Either partition both numbers and recombine or partition the second number only e.g.</p> <p> $55 + 37 = 55 + 30 + 7$ $= 85 + 7$ $= 92$ </p>  <p>55 85 92</p> <p>Adding mentally nearest multiples of 10 Continue as in stages 2 and 3 but with appropriate numbers e.g. $63 + 29$ is the same as $63 + 30 - 1$</p> <p>PENCIL AND PAPER PROCEDURES up to 4-digit numbers including decimals in the context of money</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 20px;">358</td> <td style="text-align: right;">£ 2.50</td> </tr> <tr> <td style="text-align: right;">+ 73</td> <td style="text-align: right;">+ £ 1.75</td> </tr> <tr> <td style="text-align: right; border-top: 1px solid black;">431</td> <td style="text-align: right; border-top: 1px solid black;">£ 4.25</td> </tr> <tr> <td style="text-align: right;">11</td> <td style="text-align: right;">1</td> </tr> </table>	358	£ 2.50	+ 73	+ £ 1.75	431	£ 4.25	11	1	<p>Addition Stage 5</p> <p>Partition into hundreds, tens and ones and recombine Either partition both numbers and recombine or partition the second number only e.g.</p> <p> $358 + 73 = 358 + 70 + 3$ $= 428 + 3$ $= 431$ </p>  <p>358 428 431</p> <p>Adding mentally nearest multiples of 10 or 100 Continue as in stages 2, 3 and 4 but with three digit and two digit numbers e.g. $458 + 79 = 458 + 80 - 1$</p> <p>PENCIL AND PAPER PROCEDURES</p> <p>Standard method with more than 4 digits</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 20px;">13 587</td> <td></td> </tr> <tr> <td style="text-align: right;">+ 9 675</td> <td></td> </tr> <tr> <td style="text-align: right; border-top: 1px solid black;">23 262</td> <td></td> </tr> <tr> <td style="text-align: right;">1111</td> <td></td> </tr> </table>	13 587		+ 9 675		23 262		1111		<p>Addition Stage 6</p> <p>Partition into hundreds, tens, ones and decimal fractions and recombine Either partition both numbers and recombine or partition the second number only e.g.</p> <p> $35.8 + 7.3 = 35.8 + 7 + 0.3$ $= 42.8 + 0.3$ $= 43.1$ </p>  <p>35.8 42.8 43.1</p> <p>Add the nearest multiple of 10, 100 or 1000, then adjust Continue as in stages 2, 3, 4 and 5 but with decimal numbers including extending to adding 0.9, 1.9, 2.9 etc</p> <p>PENCIL AND PAPER PROCEDURES Extend to numbers with any number of digits and decimals with 1 and 2 decimal places.</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 20px;">124.90</td> <td></td> </tr> <tr> <td style="text-align: right;">+ 117.25</td> <td></td> </tr> <tr> <td style="text-align: right; border-top: 1px solid black;">242.15</td> <td></td> </tr> <tr> <td style="text-align: right;">1 1</td> <td></td> </tr> </table>	124.90		+ 117.25		242.15		1 1	
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Subtraction Stage 1

Pictures and symbols
Sam had 10 marbles. He gave 4 to his friend. How many marbles has he got left?

- = signs and missing numbers
 $7 - 3 = \square$

Number lines (numbered) between 10 and 20

$11 - 7$ (Counting back)

The difference between 7 and 11 (Counting up)

Subtraction Stage 2

Use known number facts and place value to subtract
(partition second number only)

$$37 - 12 = 37 - 10 - 2$$

$$= 27 - 2$$

$$= 25$$

Find a small difference by counting up

 $42 - 39 = 3$

Subtract 9 or 11. Begin to add/subtract 19 or 21

 $35 - 9 = 26$

Subtraction Stage 3

Use known number facts and place value to subtract
Continue as in stage 2 but with appropriate numbers e.g.

 $97 - 15 = 97 - 10 = 87$
 $87 - 5 = 82$

Subtract mentally a 'near multiple of 10' to or from a two-digit number

Continue as in stage 2 but with two digit numbers e.g. $78 - 49$ is the same as $78 - 50 + 1$

Introduce PENCIL AND PAPER PROCEDURES

$$\begin{array}{r} 97 \\ - 15 \\ \hline 82 \end{array}$$

Subtraction Stage 4

Use known number facts and place value to subtract
Continue as in stage 3 but with appropriate numbers e.g.

 $92 - 15 = 77$

Find a small difference by counting up

Find a small difference by counting up on a number line
e.g. $5003 - 4996 = 7$

PENCIL AND PAPER PROCEDURES up to 4-digit numbers

$$\begin{array}{r} 81 \\ 9/2 \\ - 15 \\ \hline 77 \end{array}$$

Use the word **exchange** e.g. "I exchange one ten for ten ones."

Subtraction Stage 5

Use known number facts and place value to subtract numbers, including decimals

 $6.1 - 0.4 = 5.7$

Find a difference by counting up
e.g. $754 - 286 = 468$

Subtract the nearest multiple of 10 or 100, then adjust.
Continue as in stages 2, 3 and 4 but with appropriate numbers.

Subtraction Stage 6

Find a difference by counting up
e.g. $0.5 - 0.31 = 0.19$

PENCIL AND PAPER PROCEDURES
 $6467 - 2684 = 3783$

Standard method
Extend to numbers with any number of digits and decimals with 1 and 2 decimal places.

$$\begin{array}{r} 8 \quad 12 \quad 1 \\ 9 \quad 3 \quad 2 \\ - 4 \quad 5 \quad 7 \\ \hline 4 \quad 7 \quad 5 \end{array}$$

PENCIL AND PAPER PROCEDURES

Standard method with more than 4 digits.

$$\begin{array}{r}
 ^8 ^{12} ^1 \\
 932 \\
 - 457 \\
 \hline
 475
 \end{array}$$

Multiplication Stage 1

Pictures and symbols

3 marbles are in one bag. How many marbles are there in 5 bags?



$3 + 3 + 3 + 3 + 3$

5×3

Use of the following vocabulary :

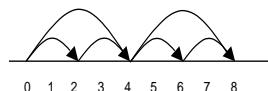
- groups of
- lots of

x = signs and missing numbers

$\square = 7 \times 2$ $14 = \square \times 2$
 $7 \times \square = 14$ $14 = \square \times 7$
 $2 \times \square = 14$ $\square \times \nabla = 4$

Multiplication Stage 2

Arrays and repeated addition



2×4 or 4×2 or $2 + 2 + 2 + 2$ or $4 + 4$

Doubling multiples of 5 up to 50

$15 \times 2 = 30$

Partition

$15 \times 2 = 30$

$10 \times 2 + 5 \times 2$
 $20 + 10 = 30$

Introduce grid method

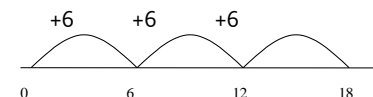
x	10	5
2	20	10

Multiplication Stage 3

Arrays and repeated addition

Understand multiplication as repeated addition and continue to use arrays and concrete objects / picture representation

Moving onto using number lines



6×3

PENCIL AND PAPER PROCEDURES

Doubling 2-digit numbers up to 50 using the Grid Method

$35 \times 2 = 70$

x	30	5
2	60	10

Leading onto multiplying 2-digit numbers by single digit

$32 \times 3 = 96$

x	30	2
3	90	6

Multiplication Stage 4

Partition

$23 \times 4 = 92$ $23 \times 4 = (20 \times 4) + (3 \times 4)$
 $= 80 + 12$

Multiplication Stage 5

Partition

$47 \times 6 = 92$ $47 \times 6 = (40 \times 6) + (7 \times 6)$
 $= (240) + (42)$

Multiplication Stage 6

Partition

$87 \times 6 = 522$ $87 \times 6 = (80 \times 6) + (7 \times 6)$
 $= 480 + 42$

$= 92$

PENCIL AND PAPER PROCEDURES

Grid method

23 x 7

x		20	3
7		140	21

Extend to HTO x O e.g. 423 x 7

Introduce formal method (short multiplication) up to HTOxO

$$\begin{array}{r}
 23 \\
 \times 7 \\
 \hline
 161
 \end{array}$$

Carrying over happens at the top

$= 282$

PENCIL AND PAPER PROCEDURES

Grid method to calculate TO x TO.

x	70	2	= 2160
30	2100	60	
8	560	16	= 576

Leading to formal method (long multiplication):

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224 \\
 11
 \end{array}$$

carrying over for the **multiplication** happens at the **top**; carrying over for the **addition** happens at the **underneath**.



$= 522$

PENCIL AND PAPER PROCEDURES

Grid method

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224 \\
 11
 \end{array}$$

x	100	20	4
20	2000	400	80
6	600	120	24

Long Multiplication

Extend to decimals with up to two decimal places.

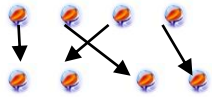
Division Stage 1

Division Stage 2

Division Stage 3

Pictures and symbols

Sam shared equally 4 marbles with his friend. How many will they both have?



Sam Tom

Use of the following vocabulary, in context:

- groups of
- lots of

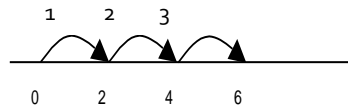
Understand division as sharing and grouping

Sharing – 6 marbles are shared between 2 people. How many do they have each?

$6 \div 2$ can be modelled as:



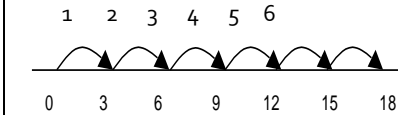
Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)



Understand division as sharing and grouping

Sharing – 18 shared between 3

Grouping - How many 3's make 18?

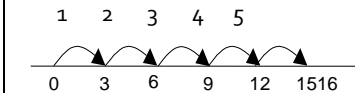


Link with multiplication

$6 \div 2 = 3$ $6 \div 3 = 2$
 $2 \times 3 = 6$ $3 \times 2 = 6$

Remainders

$16 \div 3 = 5 \text{ r}1$
 Sharing - 16 shared between 3, how many left over?



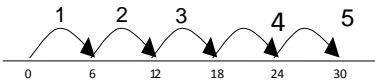
5 lots of 3; 1 is leftover

Division Stage 4

Sharing and grouping

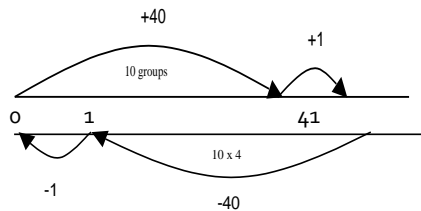
$30 \div 6$

Grouping : groups of 6 taken away and the number of groups counted e.g.



Remainders

$41 \div 4 = 10 \text{ r}1$



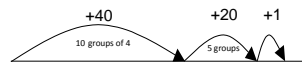
OR $41 = (10 \times 4) + 1$

Division Stage 5

Remainders with remainders

Quotients (answers) expressed as fractions or decimal fractions

$61 \div 4 = 15 \text{ r}1 = 15 \frac{1}{4}$ or 15.25



PENCIL AND PAPER PROCEDURES

Short division (with up to 4-digit numbers) depending on the question.

$7 \overline{) 98} \begin{matrix} 14 \\ 2 \end{matrix}$ $5 \overline{) 432} \begin{matrix} 86 \\ 3 \\ \text{r}2 \end{matrix}$

$86 \text{ r}2$

$86 \frac{2}{5}$

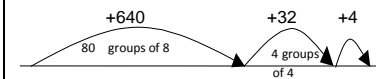
86.4

Division Stage 6

Sharing and grouping with remainders

Quotients (answers) expressed as fractions or decimal fractions

$676 \div 8 = 84.5$



PENCIL AND PAPER PROCEDURES

Long division

(including divisions of decimal numbers)

$432 \div 15 = 28 \text{ r}12 = 28 \frac{12}{15} = 28.8$

$15 \overline{) 432.0} \begin{matrix} 28 \\ \cdot 8 \\ \hline 30 \\ \downarrow \\ 132 \\ \downarrow \\ 120 \\ \downarrow \\ 120 \\ \downarrow \\ 0 \end{matrix}$